AP Calculus AB and BC - Mr. Bridges & Mr. Zwart   Name_____

Summer Hw - NO CALCULATOR

1) Dear AP Calculus AB and BC Students,

   Welcome to your first homework assignment for AP Calculus! This assignment contains ONLY review material from algebra through trigonometry and highlights concepts that are most commonly missed. Please take time to complete this assignment during the month of August so that your brain has practiced mathematics before school has started. We anticipate that this assignment will take a few hours to complete

   If you cannot remember how to do something, please use Google, Khan Academy, and old class notes for help. For example, if you forgot how to do problems 4-11, type "properties of exponents" into Google and you will find over 3 million websites offering help! We will start the year with limits, so be ready to go.

Solve the following.

2) $\dfrac{8}{5} = \dfrac{6}{x}$

3) $\dfrac{9}{4} = \dfrac{m-4}{m}$

Simplify. Your answer should contain only positive exponents.

4) $\left(2xy^{-4}\right)^3$

5) $x^3 y^4 \cdot yx^{-1}$

6) $4x^4 y^0 \cdot yx^{-3}$

7) $x^2 y^2 \cdot 4yx^{-1}$

8) $\dfrac{3xy^{-1}}{2x^2 y^{-4}}$

9) $\dfrac{3u^{-2}v^4}{2uv^{-2}}$

10) $\left(3n^4 p^3\right)^0$

11) $\left(3prq^{-2}\right)^{-1}$

Factor each completely. (Remember, NO calculator)

12) $7p^2 + 68p - 20$

13) $2x^2 - 21x + 40$

14) $9p^2 - 54p$

15) $6n^2 - 41n + 70$

16) $3b^3 - 30b^2 + 75b$

17) $5r^2 - 35r + 60$

18) $25x^2 + 260x + 315$

19) $12x^4 - 92x^3 - 144x^2$

Factor each and find all zeros. (No calculator on this entire packet.)

20) $f(x) = x^4 - 9x^2 + 8$

21) $f(x) = x^4 - 4x^3 - 5x^2$

Write the slope-intercept form of the equation.

22) through: $(4, 3)$ and $(0, -2)$

23) through: $(3, 3)$ and $(-1, 2)$

24) through: $(-2, -4)$, slope $= \dfrac{3}{2}$

25) through: $(-5, 2)$, parallel to $y = -\dfrac{3}{5}x + 2$

26) In the context of calculus, what does the word "normal" mean?

27) Given $f(x) = -x^2 + 2x - 7$ Find the equation for the line at $x = -2$ with $m = 3$.

Find the average rate of change on the given interval. (Google average rate of change if needed.)

28) $f(x) = \dfrac{1}{x + 2}$ on the interval $[-1, 2]$

Identify the points of discontinuity, holes, vertical asymptotes, horizontal asymptote, and domain of each. (Remember, no calculator allowed on this entire packet.)

29) $f(x) = \dfrac{x+3}{x^2-4}$

30) $f(x) = \dfrac{x^2-6x+8}{-3x^2+9x-6}$

31) $f(x) = \dfrac{x^2+2x-8}{4x+8}$

32) $f(x) = \dfrac{x^2-3x}{-4x^3+16x}$

33) Patrick finds the process of finding vertical asymptotes, horizontal asymptotes, and holes very confusing. Using the 4 problems above and any resources you find online, inform Patrick on how to find vertical asymptotes, horizontal asymptotes, and holes.

For each question, write each problem in an equivalent form. (In order to integrate and differentiate, Calculus will require you to be able to write expressions in equivalent forms.)

34) For example, $\sqrt[3]{x^2}$ is equivalent to $x^{\frac{2}{3}}$

35) $\sqrt[5]{x}$

36) $\cos^2 x$

37) $\sin^5 x$

38) $\sin^2 x + \cos^2 x$

39) $\dfrac{1}{\csc x}$

40) $\dfrac{x^3}{5}$

41) $\dfrac{1}{x}$

42) $(x+2)^2$

43) $ln^9 x$

Rewrite each equation in exponential form.

44) $\ln x = \dfrac{19}{10}$

45) $\log_u v = -\dfrac{9}{5}$

Condense each expression to a single logarithm.

46) $5\ln a + 15\ln b$

47) $4\ln x + 4\ln z - 12\ln y$

Expand each logarithm.

48) $\ln \dfrac{u^5}{v^2}$

49) $\ln \left(x \cdot y \cdot z^4 \right)$

Evaluate each function.

50) $k(a) = 4^a$; Find $k(-1)$

51) $f(x) = x^2 - 5$; Find $f(b)$

52) $f(x) = x^2 - 2x$; Find $f(x + h)$

53) $f(x) = 3x^2 - x$ Find $\dfrac{f(x + h) - f(x)}{h}$

Perform the indicated operation.
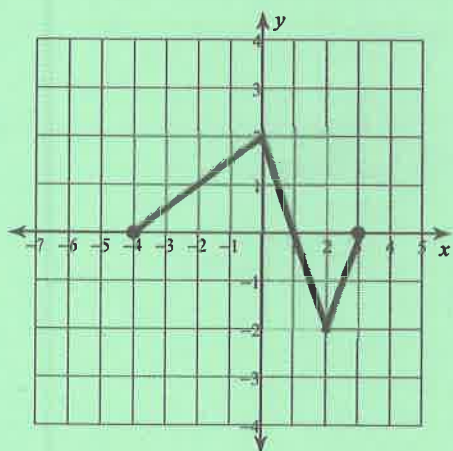
54) $f(x) = x^2 + 3x$
$g(x) = 3x - 4$
Find $f(g(x))$

55) $h(x) = 3x + 2$
$g(x) = x^3 + 3x$
Find $h(g(-2))$

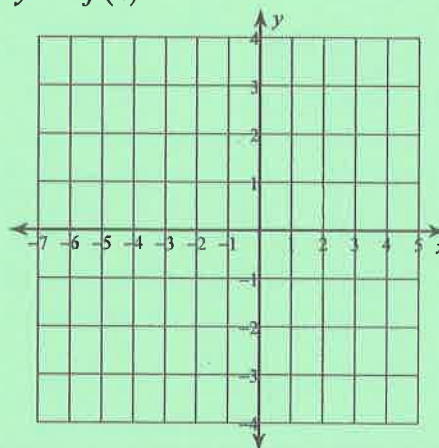56) $g(n) = 3n - 4$
Find $(g \circ g)(7)$

57) $g(x) = 2x - 3$
$h(x) = 3x + 2$
Find $g(b + 3) + h(b + 3)$

Given the graph of $y = f(x)$ in # 58 below, sketch the following graphs:
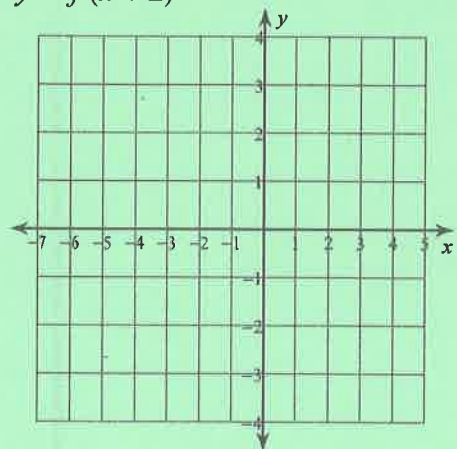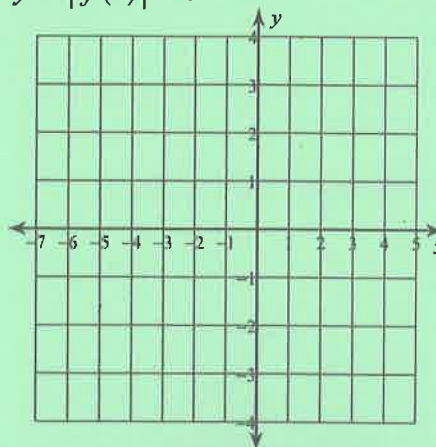
58)

59) $y = 2f(x)$

60) $y = -f(x)$
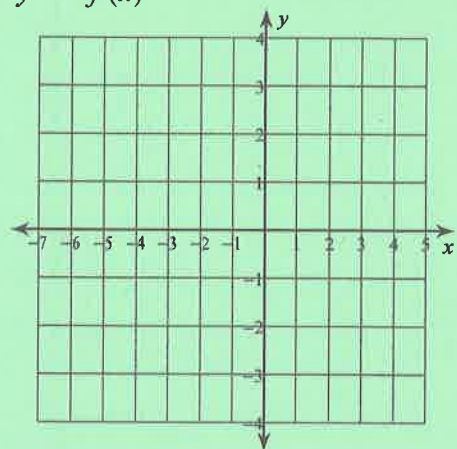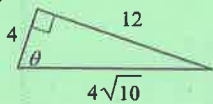
61) $y = f(x-1)$
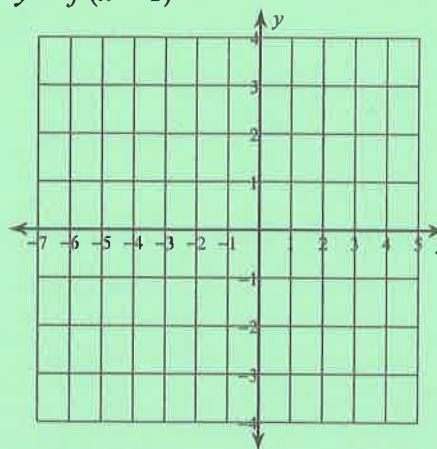
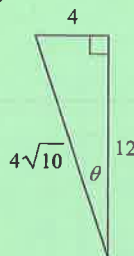62) $y = f(x+2)$

63) $y = |f(x)| + 1$

Find the value of the trig function indicated.

64) $\sin \theta$

65) $\sec \theta$

Find the exact value of each trigonometric function. (Remember, no calculator.)

66) $\sin 0$

67) $\cos \dfrac{5\pi}{3}$

68) $\sec -\pi$

69) $\sin \dfrac{\pi}{2}$

70) $\sec \dfrac{\pi}{2}$

71) $\cos -\dfrac{3\pi}{4}$

72) $\tan -\dfrac{\pi}{3}$

73) $\cot \dfrac{\pi}{4}$

Solve each equation for $0 \le \theta < 2\pi$.
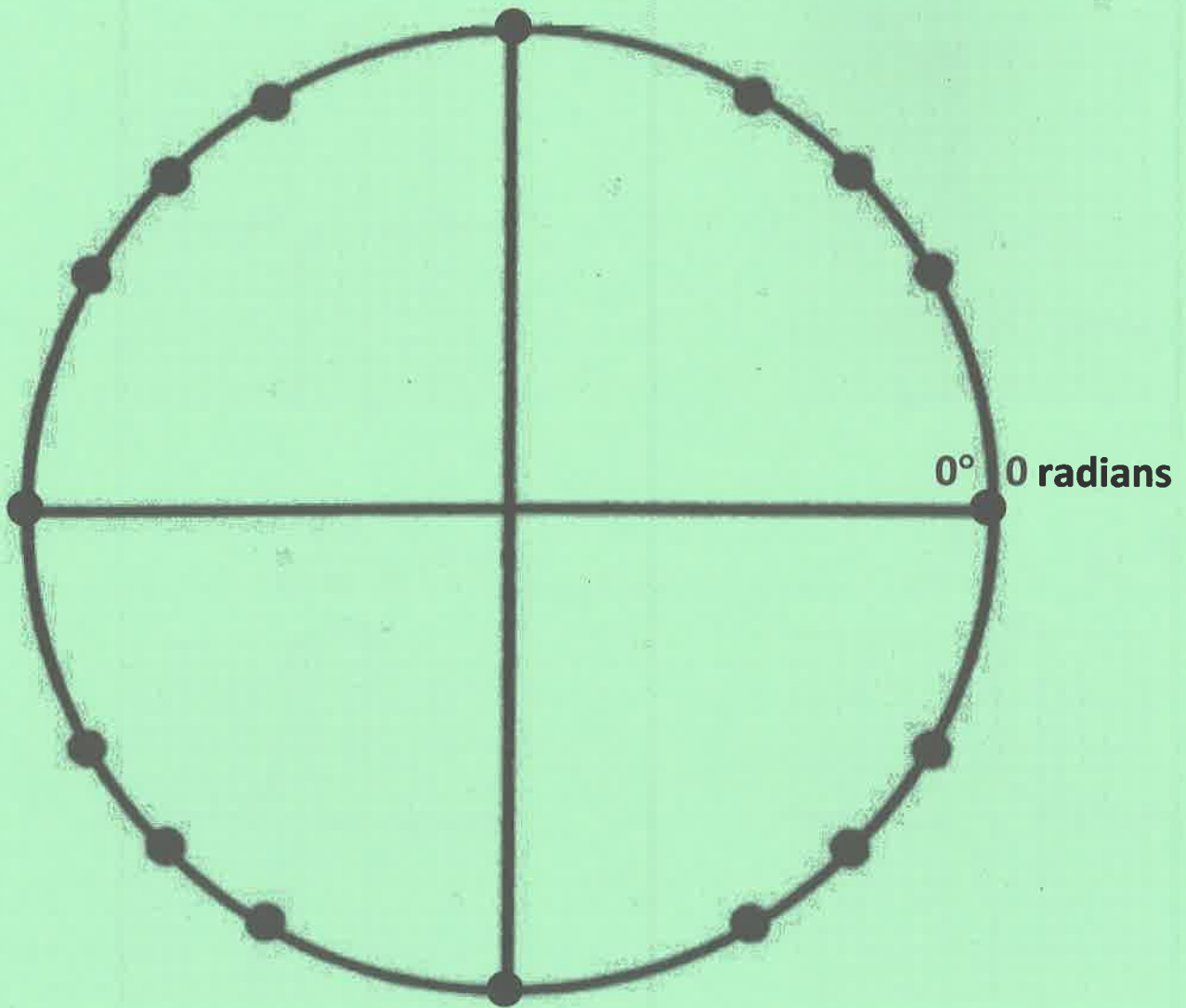
74) $\sin \theta = 0$

75) $\tan \theta = -1$

76)

Trigonometry is an important part of AP Calculus. You are expected to know how to evaluate trig functions for radian values found on the unit circle.
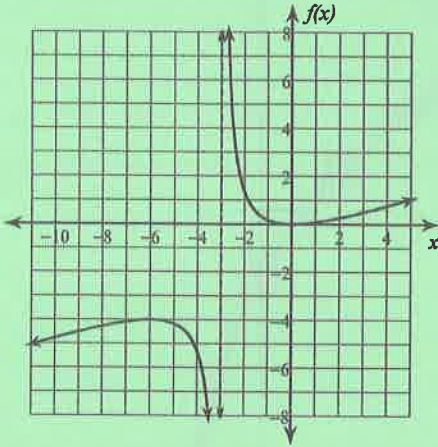
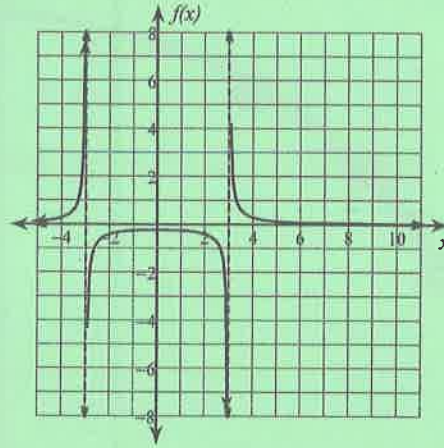**Try to fill out this unit circle from memory. Memorize these values!**

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Summer Hw Part II No Calculator Allowed

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Evaluate each limit.

77) $\lim\limits_{x \to -3^-} \dfrac{x^2}{3x+9}$

78) $\lim\limits_{x \to 3} \dfrac{2}{x^2-9}$

79) $\lim\limits_{x \to 3^-} -\dfrac{x-3}{x^2-5x+6}$
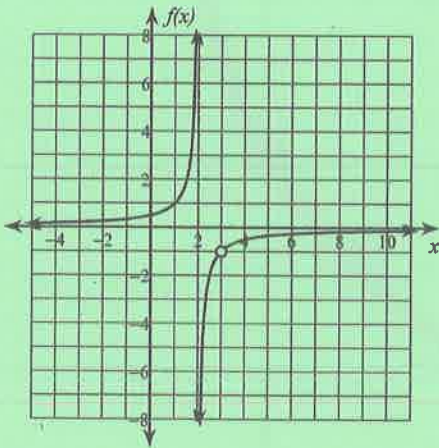
80) $\lim\limits_{x \to 3} \dfrac{x-2}{x^2-6x+9}$
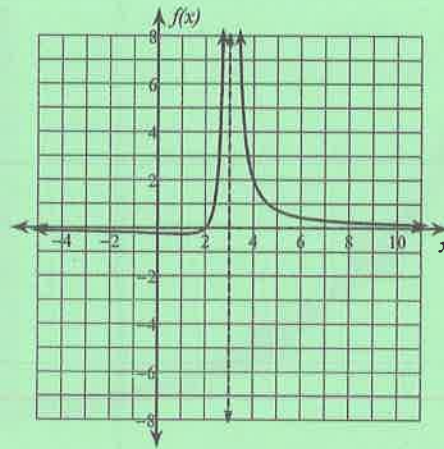
81) $\lim\limits_{t \to 1} \sqrt{t+5}$

82) $\lim\limits_{x \to 3} f(x),\ f(x) = \begin{cases} -x+1, & x<3 \\ -x^2+4x-5, & x \ge 3 \end{cases}$

Answers to Summer Hw - NO CALCULATOR

1) Please read the introduction.   2) $x = 3.75$ 3) $x = -3.2$

4) $\dfrac{8x^3}{y^{12}}$ 5) $y^5 x^2$ 6) $4yx$ 7) $4y^3 x$

8) $\dfrac{3y^3}{2x}$ 9) $\dfrac{3v^6}{2u^3}$ 10) 1   11) $\dfrac{q^2}{3pr}$

12) $(7p - 2)(p + 10)$ 13) $(2x - 5)(x - 8)$ 14) $9p(p - 6)$ 15) $(3n - 10)(2n - 7)$

16) $3b(b - 5)^2$ 17) $5(r - 4)(r - 3)$ 18) $5(5x + 7)(x + 9)$ 19) $4x^2(3x + 4)(x - 9)$

20) Factors to: $f(x) = (x^2 - 8)(x - 1)(x + 1)$
    Zeros: $\{2\sqrt{2}, -2\sqrt{2}, 1, -1\}$

21) Factors to: $f(x) = x^2(x + 1)(x - 5)$
    Zeros: $\{0 \text{ mult. } 2, -1, 5\}$

22) $y = \dfrac{5}{4}x - 2$ 23) $y = \dfrac{1}{4}x + \dfrac{9}{4}$ 24) $y = \dfrac{3}{2}x - 1$ 25) $y = -\dfrac{3}{5}x - 1$
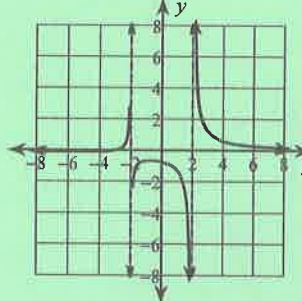
26) Normal means perpendicular.

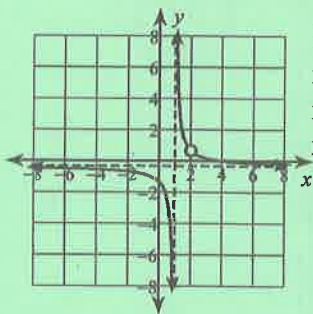27) $f(-2) = -15$ So, $y + 15 = 3(x + 2)$ Solve for y: $y = 3x - 9$

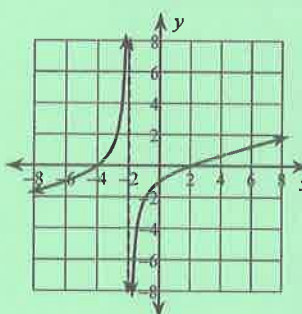28) The average rate of change is $-\dfrac{1}{4}$

29)

Discontinuities: 2, −2
Vertical Asym.: $x = 2, x = -2$
Holes: None
Horz. Asym.: $y = 0$
Domain:
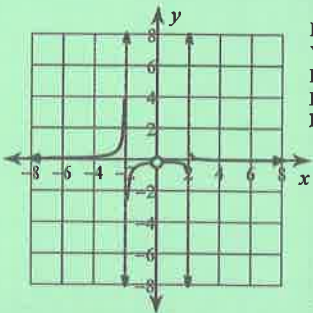 All reals except 2, −2

30)

Discontinuities: 1, 2
Vertical Asym.: $x = 1$
Holes: $x = 2$
Horz. Asym.: $y = -\dfrac{1}{3}$
Domain:
  All reals except 1, 2

31)

Discontinuities: −2
Vertical Asym.: $x = -2$
Holes: None
Horz. Asym.: None
Domain:
  All reals except −2

32)

Discontinuities: 2, −2, 0
Vertical Asym.: $x = 2, x = -2$
Holes: $x = 0$
Horz. Asym.: $y = 0$
Domain:
  All reals except 2, −2, 0

33) This one is all for you to figure out! Look at the above examples and try using Google if you need to!

34) This is an example problem.

35) $x^{\frac{1}{5}}$

36) $\cos^2 x = \cos x \cos x = (\cos x)^{\wedge}2$ 37) $(\sin x)^{\wedge}5$

38) 1 (Pythagorean Trig Identity)   39) $\sin x$

40) $\dfrac{1}{5}x^3$

41) x^{-1} 42) $x^2 + 4x + 4$ 43) $(\ln x)^9$

44) $e^{\frac{19}{10}} = x$

45) $u^{-\frac{9}{5}} = v$

46) $\ln \left(b^{15}a^5\right)$

47) $\ln \dfrac{z^4 x^4}{y^{12}}$

48) $5\ln u - 2\ln v$

49) $\ln x + \ln y + 4\ln z$

50) $\dfrac{1}{4}$

51) $b^2 - 5$

52) $x^2 + 2xh + h^2 - 2x - 2h$

53) $6x + 3h - 1$
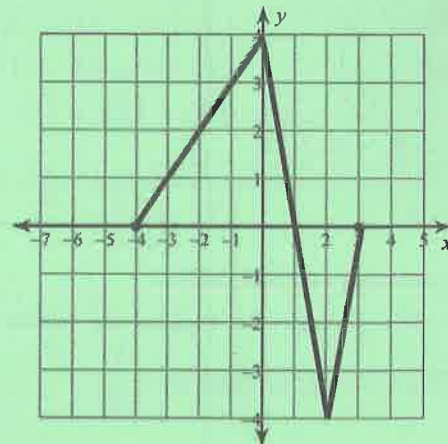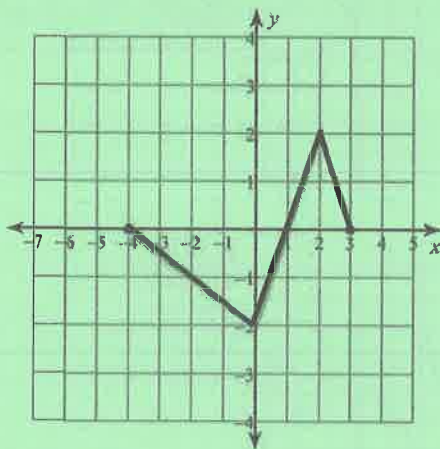
54) $9x^2 - 15x + 4$

55) -40

56) 47

57) $5b + 14$

58) The graph should be used for Questions #59-63.
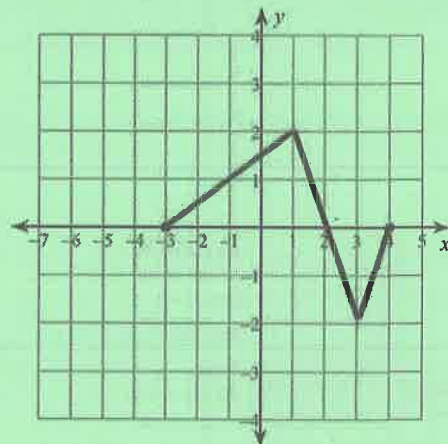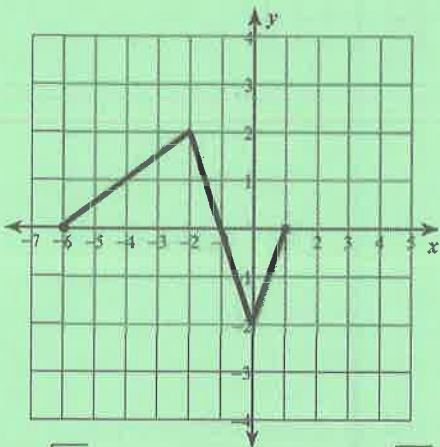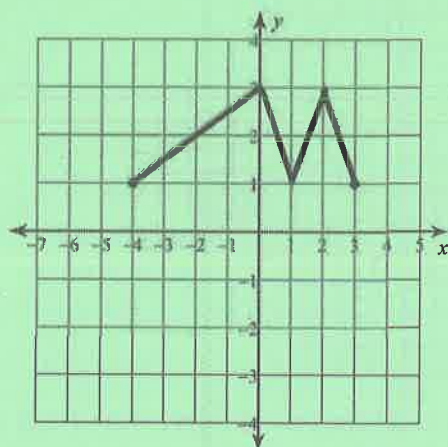
59)

60)

61)

62)

63)

64) $\dfrac{3\sqrt{10}}{10}$

65) $\dfrac{\sqrt{10}}{3}$

66) 0

67) $\dfrac{1}{2}$

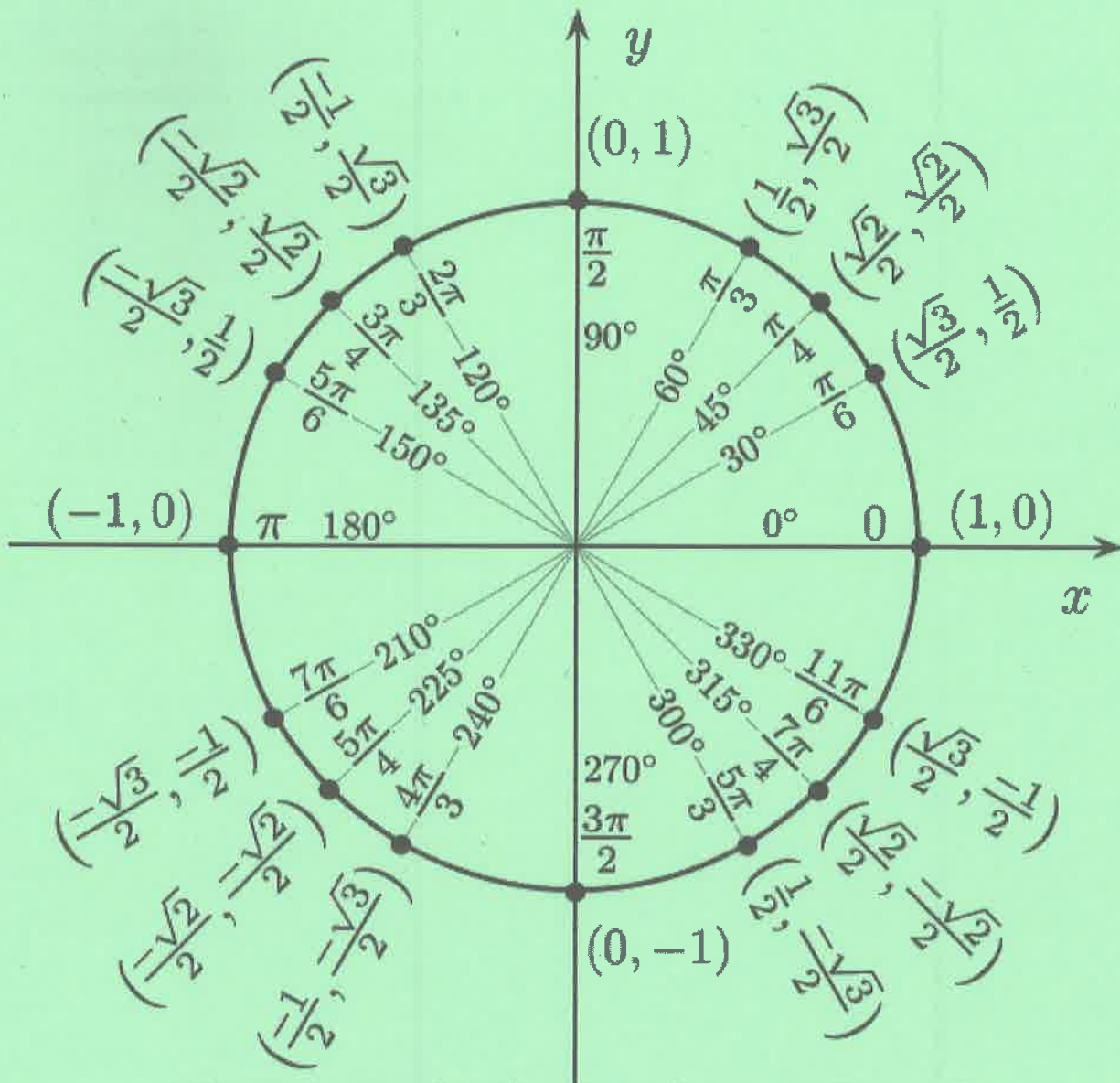68) -1

69) 1

70) Undefined

71) $-\dfrac{\sqrt{2}}{2}$

72) $-\sqrt{3}$

73) 1

74) $\{0, \pi\}$

75) $\left\{\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right\}$

Unit Circle

$(0, 1)$

$\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

$\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

$\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$\left(\dfrac{-1}{2}, \dfrac{\sqrt{3}}{2}\right)$

$\left(\dfrac{-\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

$\left(\dfrac{-\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$(-1, 0)$

$(1, 0)$

y

x

$\dfrac{\pi}{2}$ 90°

$\dfrac{2\pi}{3}$ 120°

$\dfrac{3\pi}{4}$ 135°

$\dfrac{5\pi}{6}$ 150°

π 180°

$\dfrac{\pi}{3}$ 60°

$\dfrac{\pi}{4}$ 45°

$\dfrac{\pi}{6}$ 30°

0° 0

$\dfrac{7\pi}{6}$ 210°

$\dfrac{5\pi}{4}$ 225°

$\dfrac{4\pi}{3}$ 240°

270° $\dfrac{3\pi}{2}$

$\dfrac{11\pi}{6}$ 330°

$\dfrac{7\pi}{4}$ 315°

$\dfrac{5\pi}{3}$ 300°

$\left(-\dfrac{\sqrt{3}}{2}, \dfrac{-1}{2}\right)$

$\left(\dfrac{-\sqrt{2}}{2}, \dfrac{-\sqrt{2}}{2}\right)$

$\left(\dfrac{-1}{2}, \dfrac{-\sqrt{3}}{2}\right)$

$\left(\dfrac{\sqrt{3}}{2}, \dfrac{-1}{2}\right)$

$\left(\dfrac{\sqrt{2}}{2}, \dfrac{-\sqrt{2}}{2}\right)$

$\left(\dfrac{1}{2}, \dfrac{-\sqrt{3}}{2}\right)$

$(0, -1)$

Inverse function	Domain	Range
$\theta = \sin^{-1} x$	$-1 \le x \le 1$	$-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$
$\theta = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$\theta = \tan^{-1} x$	Real numbers	$-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$
$\theta = \operatorname{cosec}^{-1} x$	$x \ge 1$ or $x \le -1$	$-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2},\ \theta \ne 0$
$\theta = \sec^{-1} x$	$x \ge 1$ or $x \le -1$	$0 < \theta \le \pi,\ \theta \ne \dfrac{\pi}{2}$
$\theta = \cot^{-1} x$	Real numbers	$0 < \theta < \pi$

Answers to Summer Hw Part II No Calculator Allowed (ID: 1)

77) $-\infty$

78) Does not exist.

79) -1

80) ∞

81) $\sqrt{6}$

82) -2